Intro. to ODEs
Quiz 5 Name: ____

Instructions: Solve the following systems of linear equations by translating the system into an augmented matrix and reducing to RREF.

1)

$$\begin{aligned} x - 3y + 5z &= 6 \\ 2x + y - 3z &= -1 \\ 3x - 2y + z &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 5 & 6 \\ 2 & 1 & -3 & -1 \\ 3 & -2 & 1 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 5 & 6 \\ 0 & 7 & -13 & -13 \\ 0 & 7 & -14 & -14 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 5 & 6 \\ 0 & 7 & -13 & -13 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3 & 0 & 1 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

So, the unique solution is $(x, y, z) = (1, 0, 1)$.

2)

$$\begin{aligned} 2x + y + 2z &= 1 \\ 3x - y + z &= 2 \\ 3x + 4y + 5z &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 3 & 4 & 5 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & 2 & 1 \\ 1 & -2 & -1 & 1 \\ 3 & 4 & 5 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 2 & 1 & 2 & 1 \\ 3 & 4 & 5 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 5 & 4 & -1 \\ 0 & 10 & 8 & -2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 4/5 & -1/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3/5 & 3/5 \\ 0 & 1 & 4/5 & -1/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So, there are an infinite number of solutions which are characterized by

$$\left\{ \left(\frac{3}{5} - \frac{3}{5}z,\ -\frac{1}{5} - \frac{4}{5}z,\ z \right) \middle|\ z \text{ is free} \right\}.$$